Università di Genova | DICCA DIPARTIMENTO DI INGEGNERIA CIVILE, CHIMICA E AMBIENTALE

TURBULENT CHANNEL FLOW OVER ANISOTROPIC POROUS SUBSTRATES

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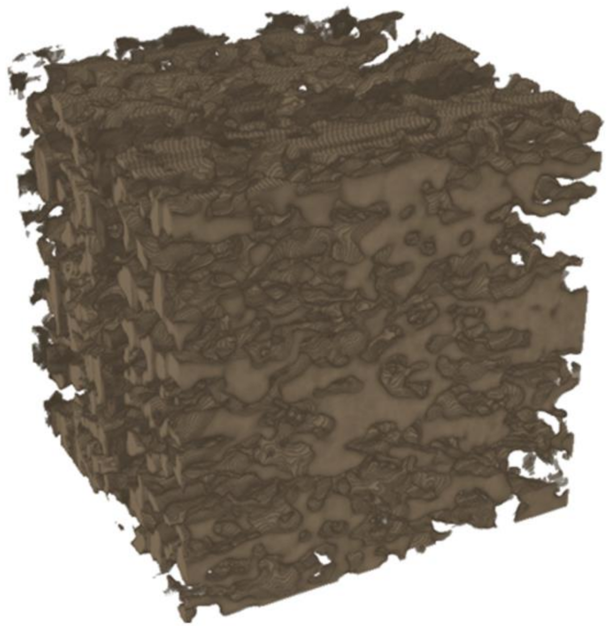
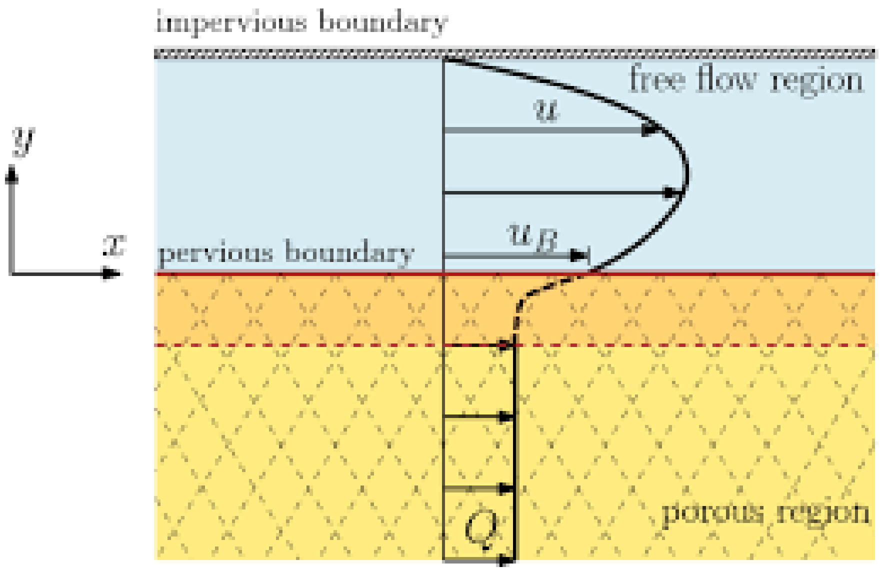


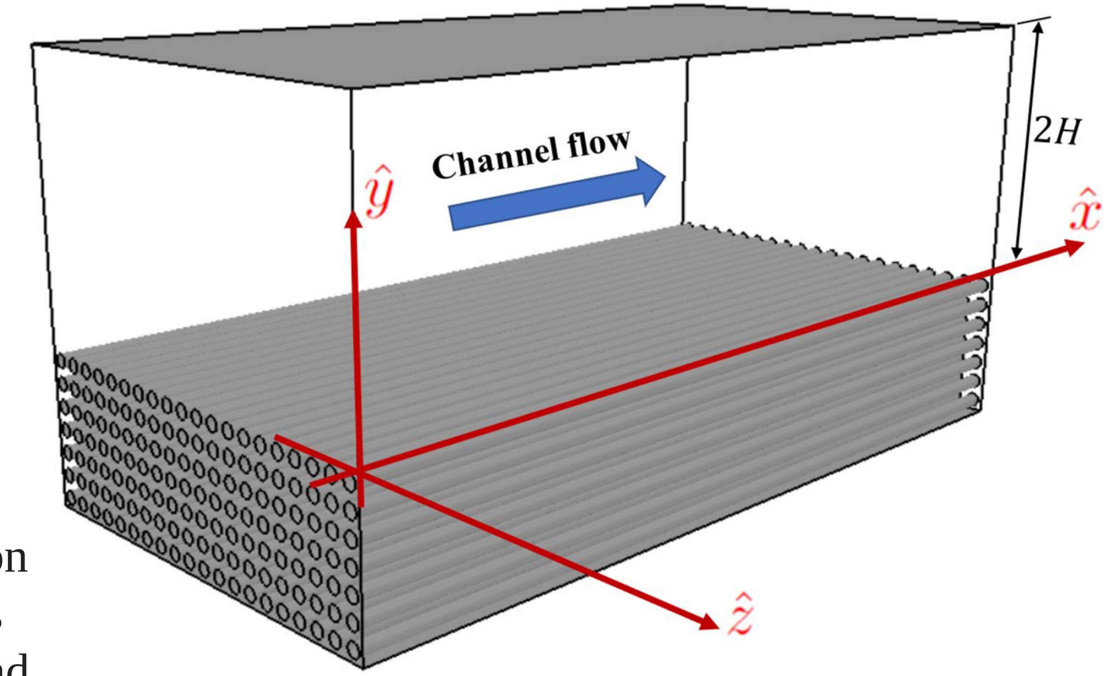


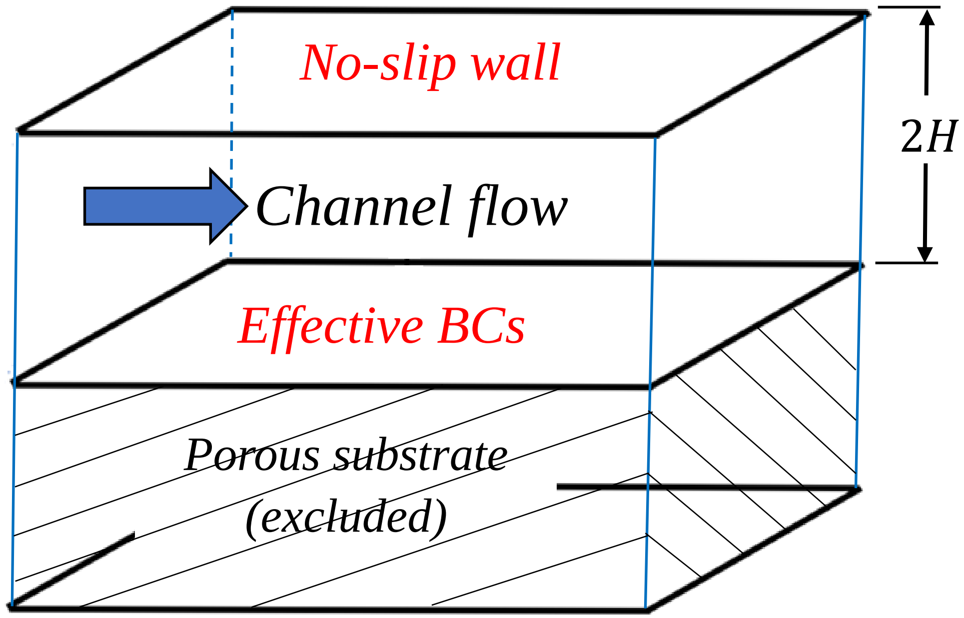
Rapidly-varying properties (related to surface heterogeneity) → Roughness, irregularity, porosity, compliance, super-hydrophobicity, etc.

Homogenization framework

Ahmed et al (2022). "A Homogenization Approach for Turbulent Channel Flows over Porous Substrates: Formulation and Implementation of Effective Boundary Conditions" *Fluids* **7**, 5: 178.

Upscaled, homogeneous properties (for *effective* boundary conditions) → Navier-slip coefficients, material permeability, interfacial permeability, etc.

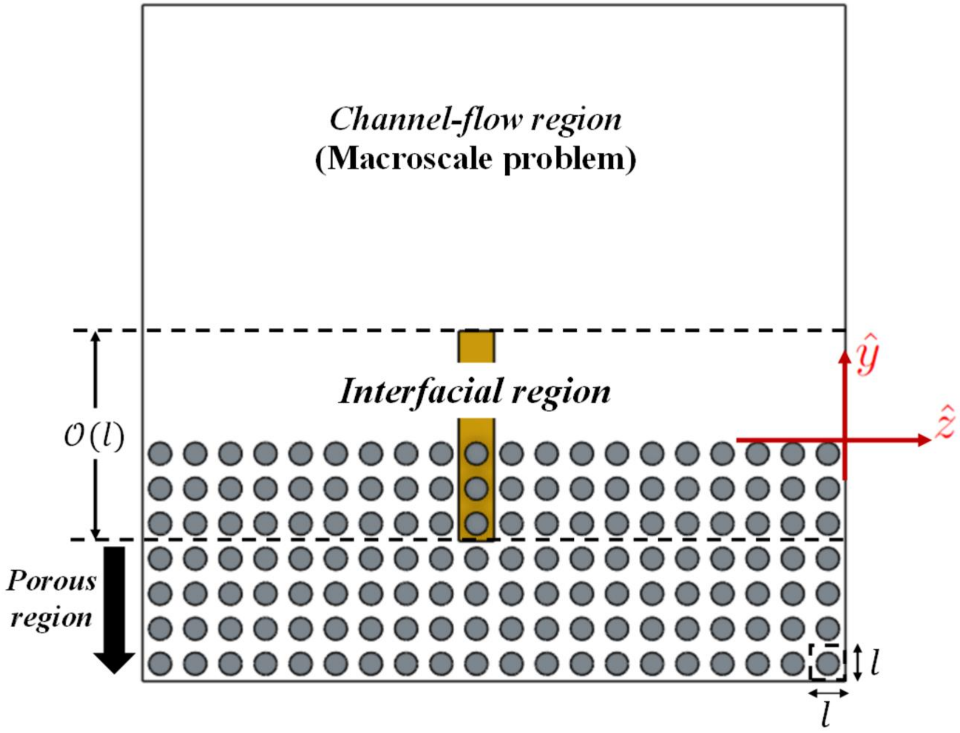


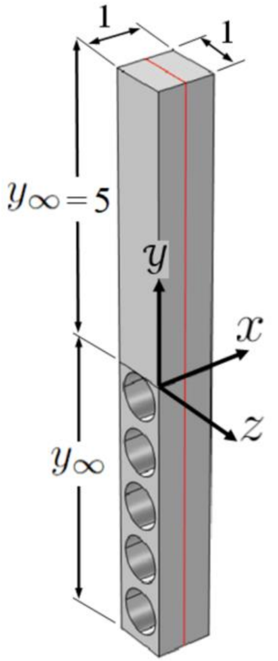


$$\epsilon = \frac{\ell}{H} \ll 1$$

$$microscopic, fast\ variable:\ x_i = \frac{\hat{x}_i}{\ell},$$

$$macroscopic, slow\ variable:\ X_i = \frac{\hat{x}_i}{H}$$





Dimensional governing equations (Incompressible Navier-Stokes) → Domain decomposition → Scaling of macroscopic and microscopic problems → Continuity conditions at the matching interface → Asymptotic expansions of microscopic variables in terms of ϵ → Problem reconstruction at different orders of ϵ → Generic forms of the solutions and definition of *ad hoc* auxiliary systems → Numerical solutions of the auxiliary systems → Estimation of solutions at the matching interface → Effect of the matching interface location → Solution Extrapolation to a matching interface at $y = 0$ → **Formal expressions of *effective* B.Cs**

Effective boundary conditions

$$U\Big|_{Y=0} = \underbrace{\epsilon\, \boldsymbol{\lambda_x}\, S_{12}\Big|_{Y=0}}_{\text{Navier-slip condition}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}^{itf}_{xy}}\, \frac{\partial S_{22}}{\partial X}\Big|_{Y=0}}_{\text{Second-order correction}} + \mathcal{O}(\epsilon^3)$$

$$S_{12} = \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X},$$

$$S_{22} = -Re\, P + 2\frac{\partial V}{\partial Y},$$

$$S_{32} = \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z}.$$

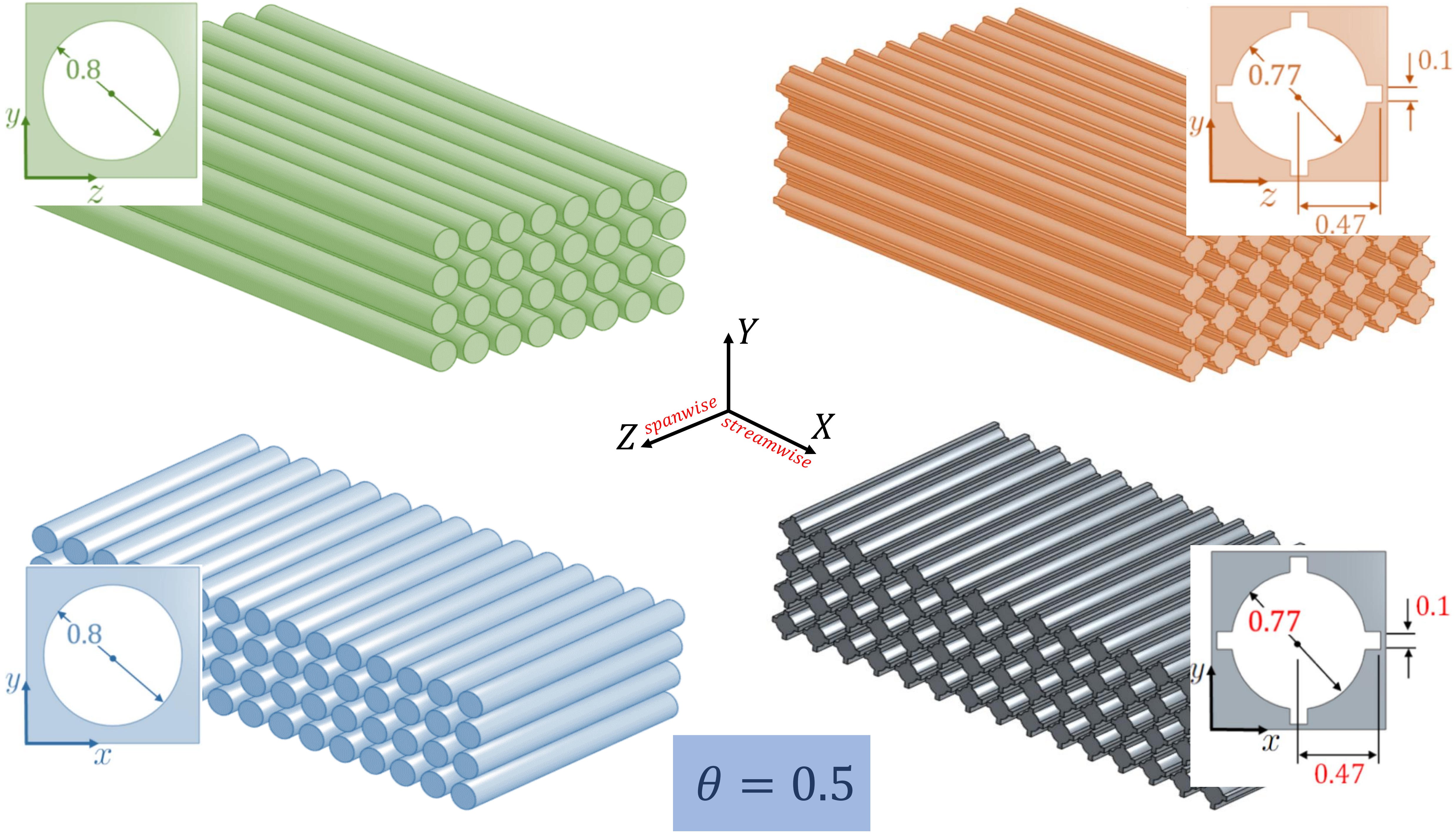
$$V\Big|_{Y=0} = \underbrace{-\epsilon^2\, \boldsymbol{\mathcal{K}^{itf}_{xy}}\, \frac{\partial S_{12}}{\partial X}\Big|_{Y=0} - \epsilon^2\, \boldsymbol{\mathcal{K}^{itf}_{zy}}\, \frac{\partial S_{32}}{\partial Z}\Big|_{Y=0}}_{\text{Second-order, effect of interface permeabilities}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}_{yy}}\, \frac{\partial S_{22}}{\partial Y}\Big|_{Y=0}}_{\text{Second-order, effect of medium permeability}} + \mathcal{O}(\epsilon^3)$$

$$W\Big|_{Y=0} = \underbrace{\epsilon\, \boldsymbol{\lambda_z}\, S_{32}\Big|_{Y=0}}_{\text{Navier-slip condition}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}^{itf}_{zy}}\, \frac{\partial S_{22}}{\partial Z}\Big|_{Y=0}}_{\text{Second-order correction}} + \mathcal{O}(\epsilon^3)$$

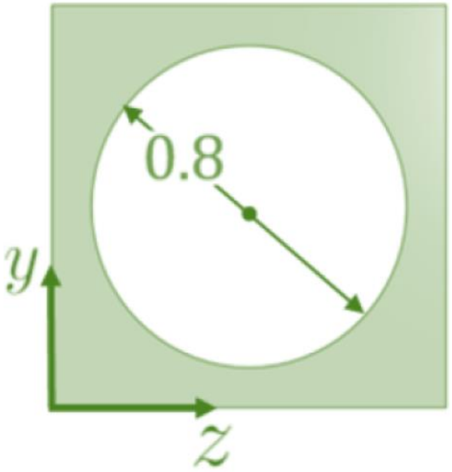
$\boldsymbol{\lambda_{x,z}}$: Navier-slip coefficients

$\boldsymbol{\mathcal{K}^{itf}_{xy,zy}}$: Interface permeability coefficents

$\boldsymbol{\mathcal{K}_{yy}}$: medium permeability

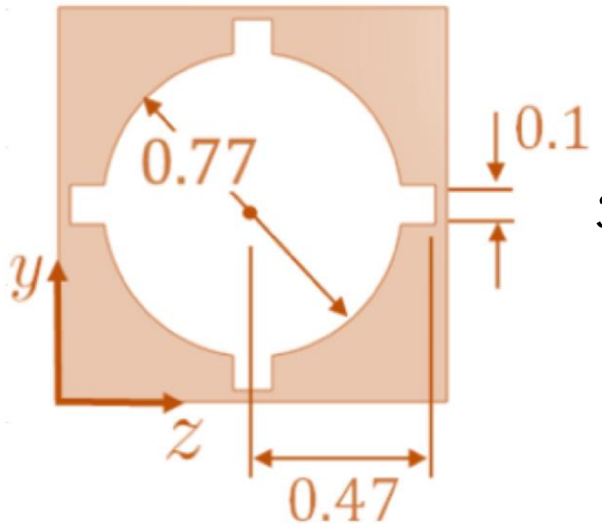

0.8
y
z
0.77
0.1
0.47
Y
Z
X
spanwise
streamwise
0.8
y
x
0.77
0.1
0.47
y
x
θ = 0.5

Longitudinal cylinders (*LC*)



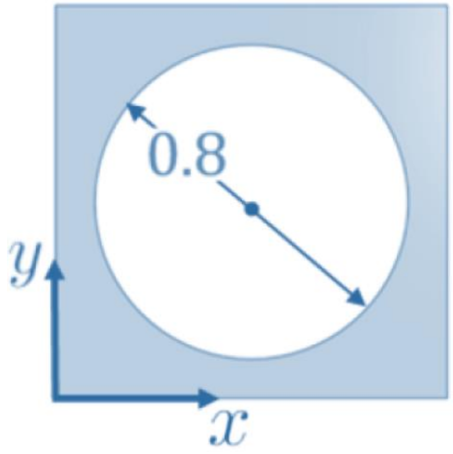
$$\lambda_x = 0.0688, \quad \lambda_z = 0.0451,$$
$$\mathcal{K}_{xy}^{itf} = 0.0056, \; \mathcal{K}_{zy}^{itf} = 0.0022,$$
$$\mathcal{K}_{yy} = 0.0018.$$

Longitudinal modified cylinders (*LM*)



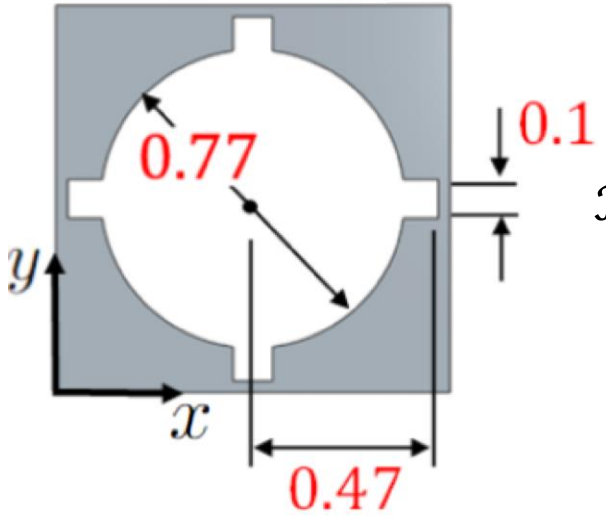
$$\lambda_x = 0.1130, \quad \lambda_z = 0.0590,$$
$$\mathcal{K}_{xy}^{itf} = 0.0121, \; \mathcal{K}_{zy}^{itf} = 0.0041,$$
$$\mathcal{K}_{yy} = 0.00012.$$

Transverse cylinders (*TC*)

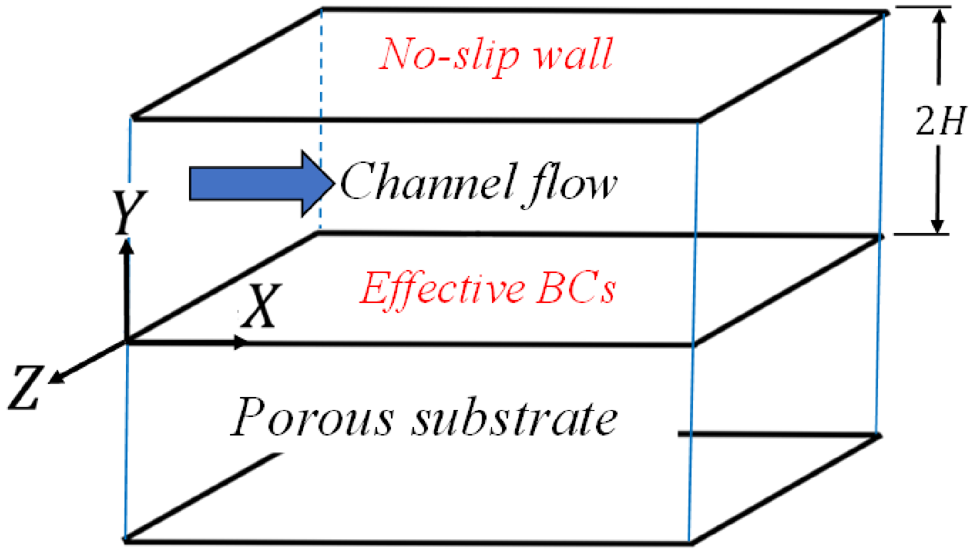


$$\lambda_x = 0.0451, \quad \lambda_z = 0.0688,$$
$$\mathcal{K}_{xy}^{itf} = 0.0022, \; \mathcal{K}_{zy}^{itf} = 0.0056,$$
$$\mathcal{K}_{yy} = 0.0018.$$

Transverse modified cylinders (*TM*)



$$\lambda_x = 0.0590, \quad \lambda_z = 0.1130,$$
$$\mathcal{K}_{xy}^{itf} = 0.0041, \; \mathcal{K}_{zy}^{itf} = 0.0121,$$
$$\mathcal{K}_{yy} = 0.00012.$$



τ_0: Total shear stress at $Y = 0$

τ_2: Total shear stress at $Y = 2$

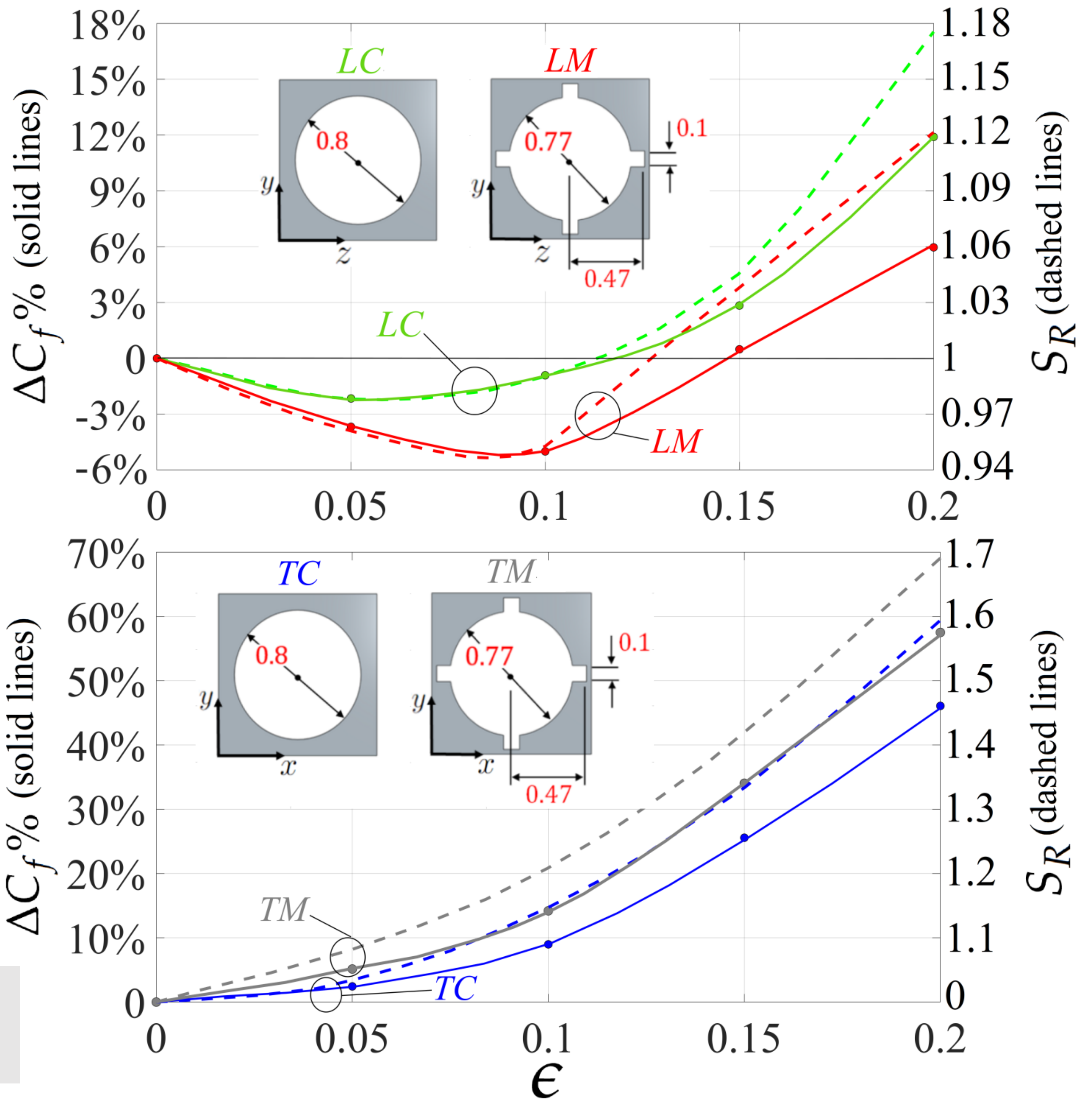
$$S_R = \frac{\tau_0}{\tau_2}$$

$\tau_M = \frac{\Delta P}{L_X} H = \frac{\tau_0 + \tau_2}{2}$ is the bulk shear stress

$$u_\tau = \sqrt{\frac{\tau_M}{\rho}}, \qquad Re_\tau = \frac{\rho u_\tau H}{\mu} = 193,$$

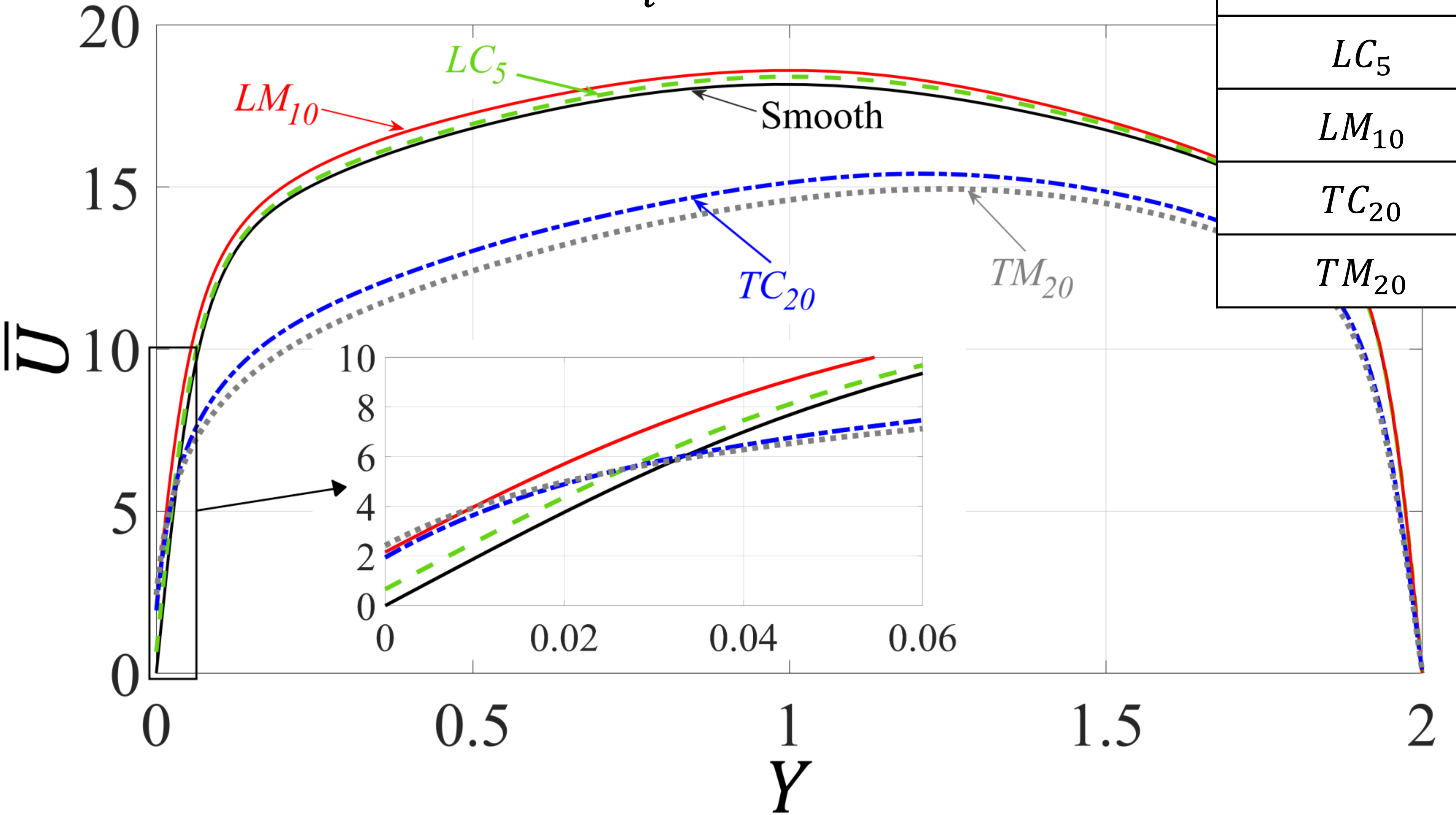
$$C_f = \frac{2\tau_M}{\rho\, U_{bulk}^2}$$

$$\Delta C_f\% = \frac{C_{f,porous} - C_{f,smooth}}{C_{f,smooth}} \times 100(\%)$$



Mean velocity profiles (in global coordinates)

$$\bar{U} = \frac{\overline{(\hat{u})}}{u_\tau}, \qquad Y = \frac{\hat{y}}{H}$$

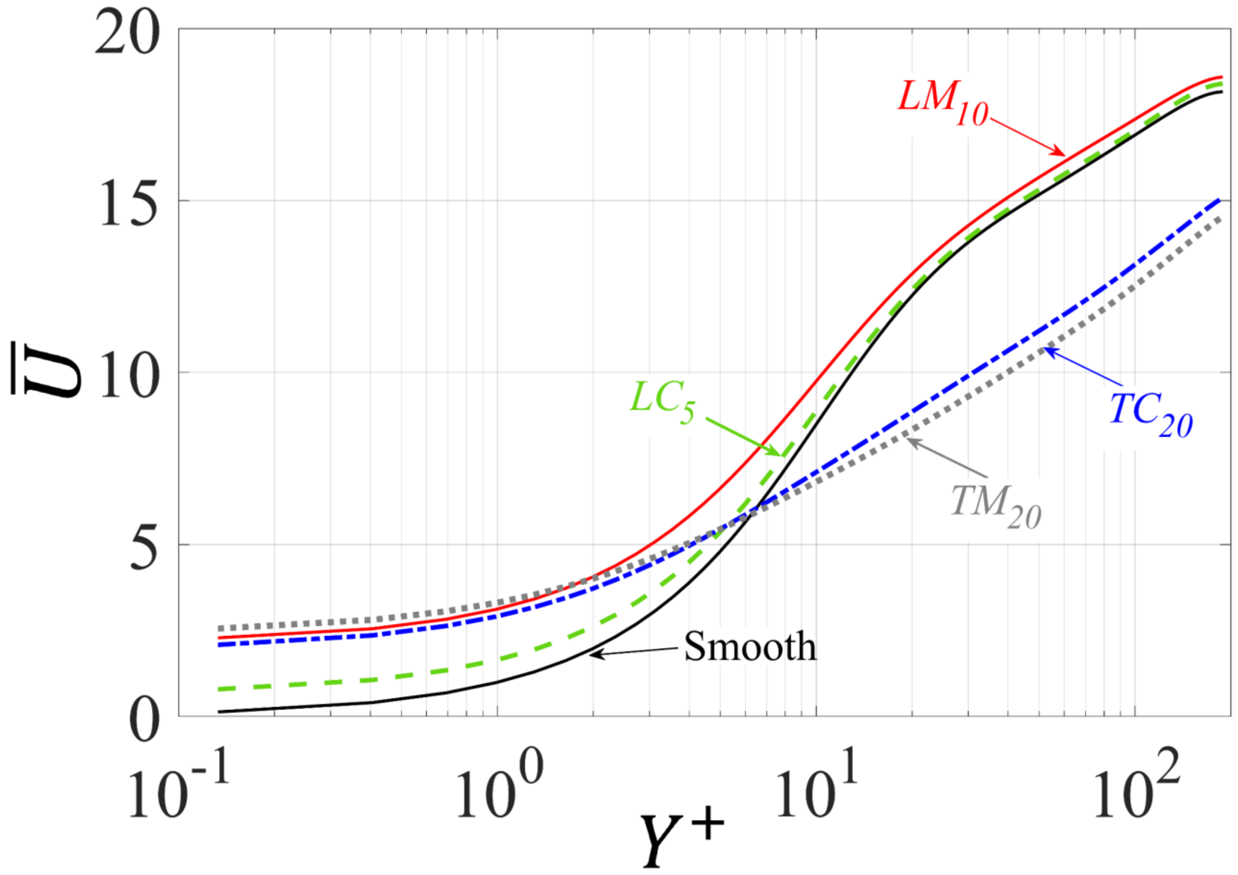


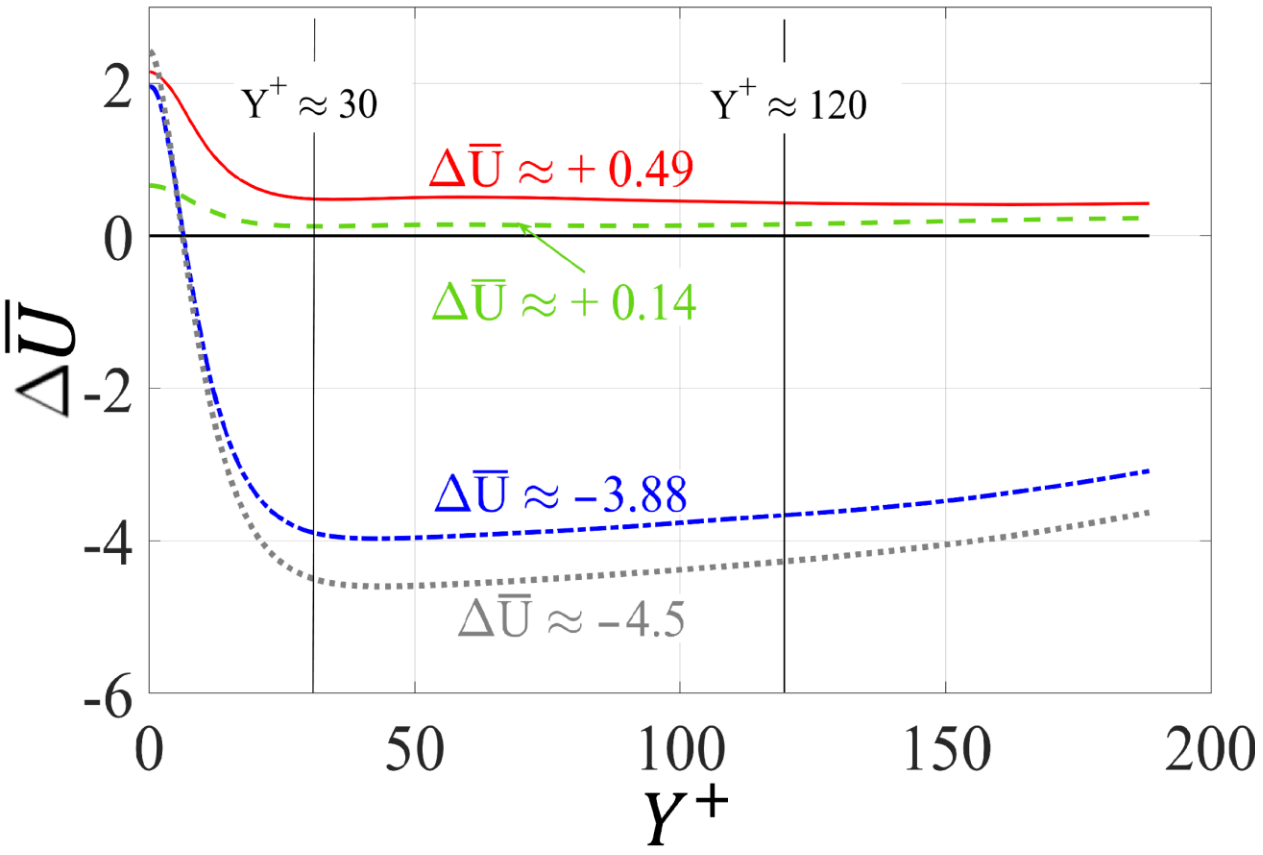
Configuration	λ_x^+	$\bar{U}\big\rvert_{Y=0}$
LC_5	0.6539	0.6605
LM_{10}	2.1470	2.1573
TC_{20}	1.7149	1.9370
TM_{20}	2.2420	2.4168

$$Re_\tau = \frac{\rho u_\tau H}{\mu} = 193$$

Mean velocity profiles (in wall coordinates)

$$\overline{U} = \frac{\overline{(\hat{u})}}{u_\tau}, \qquad Y^+ = \frac{\rho u_\tau \hat{y}}{\mu} = Y \times Re_\tau$$

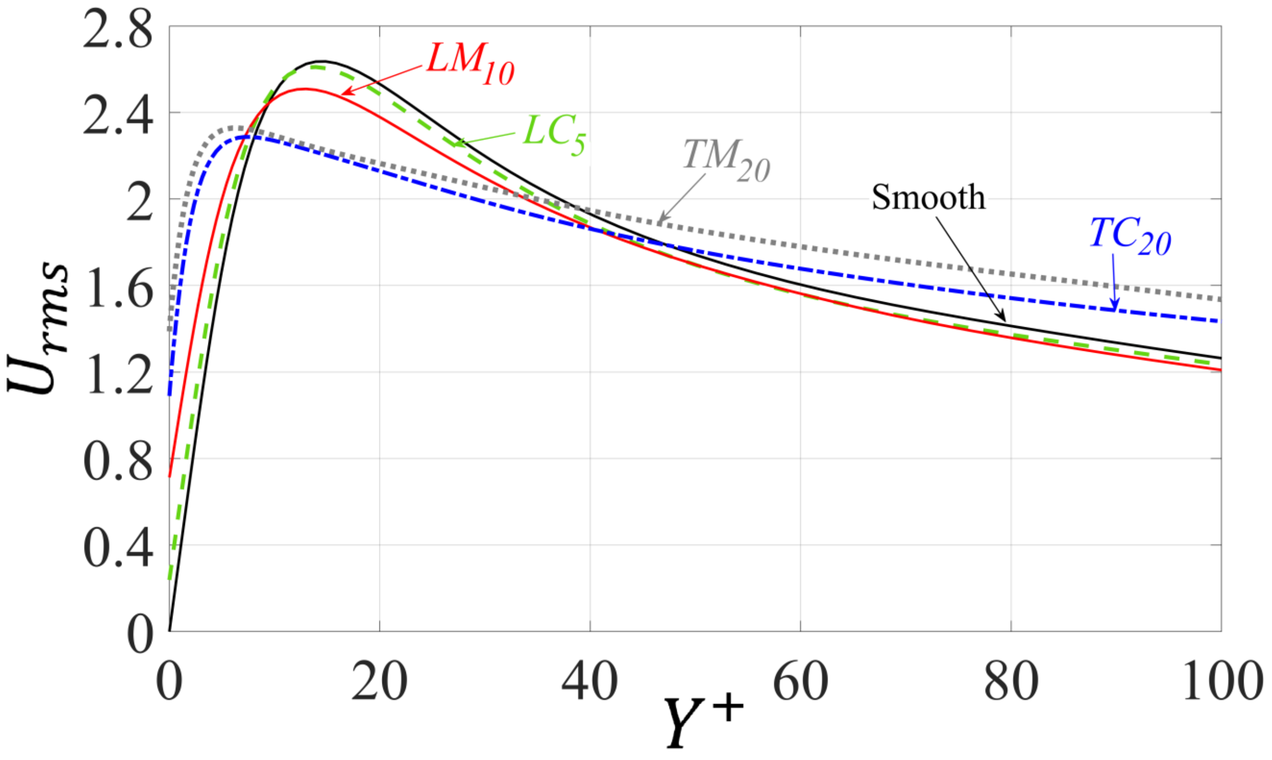


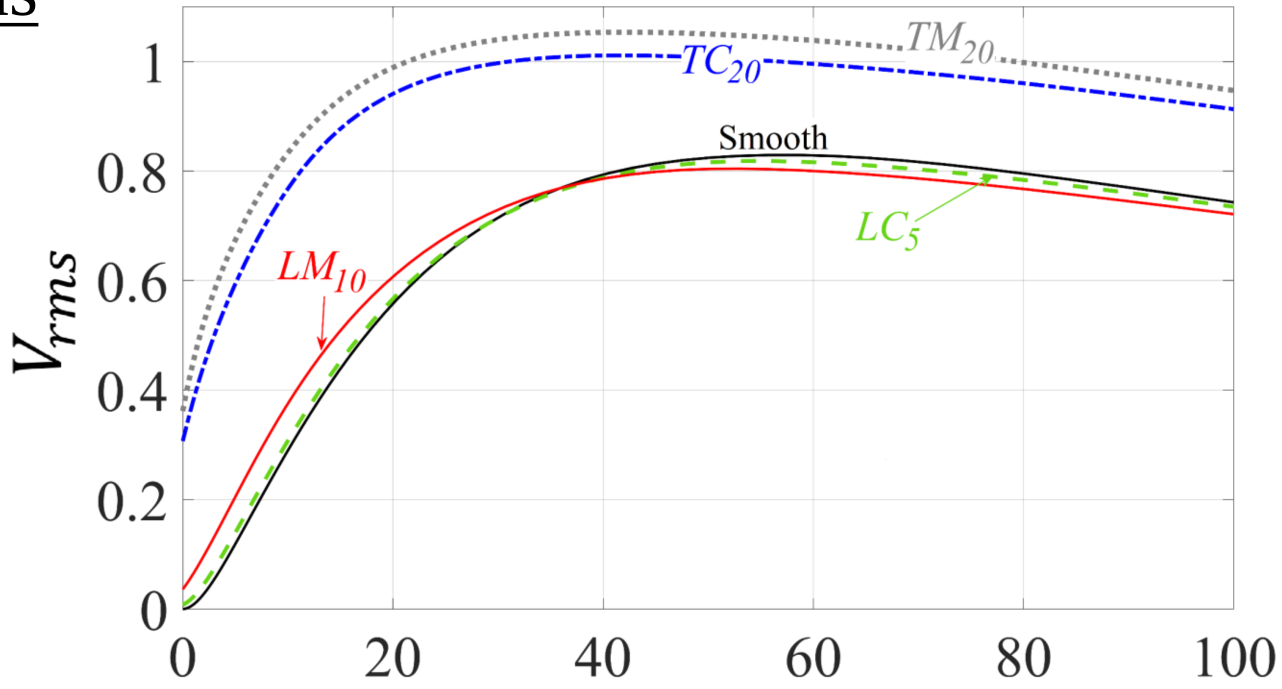


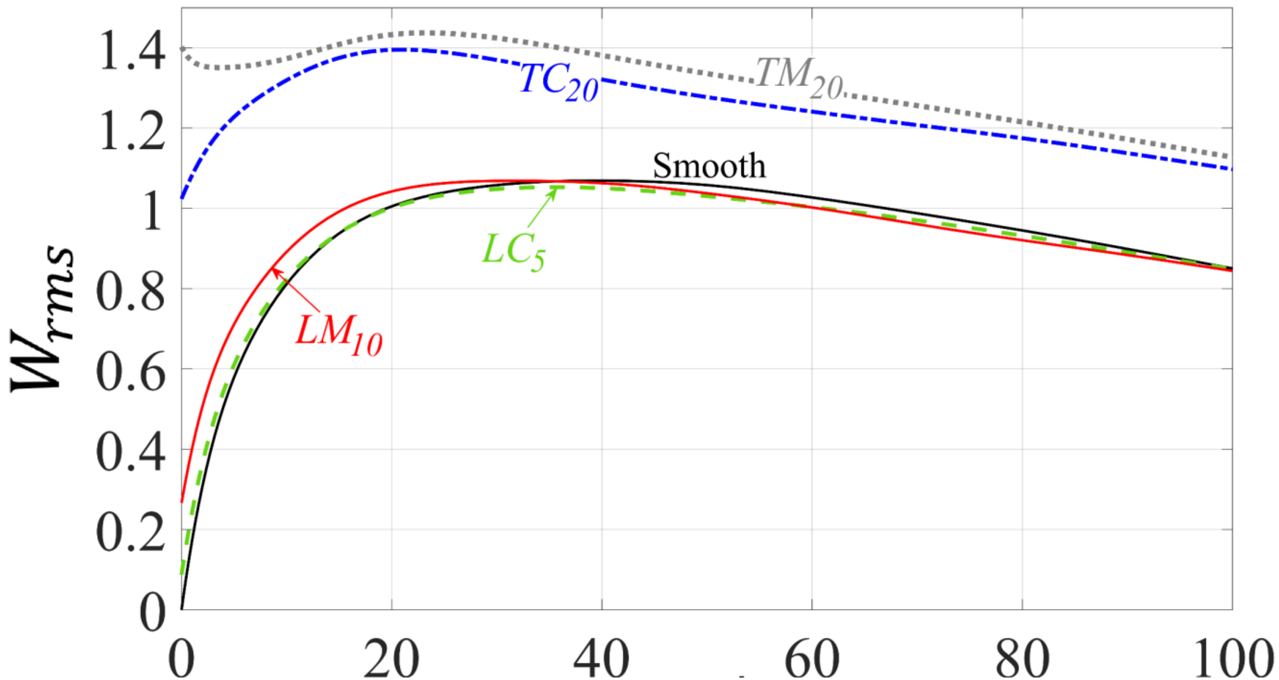
	Smooth	LC_5	LM_{10}	TC_{20}	TM_{20}
$\Delta\overline{U}$	0	+0.14	+0.49	−3.88	−4.50
$\Delta C_f\%$	0	-2.157%	-5.005%	+46.078	+57.513

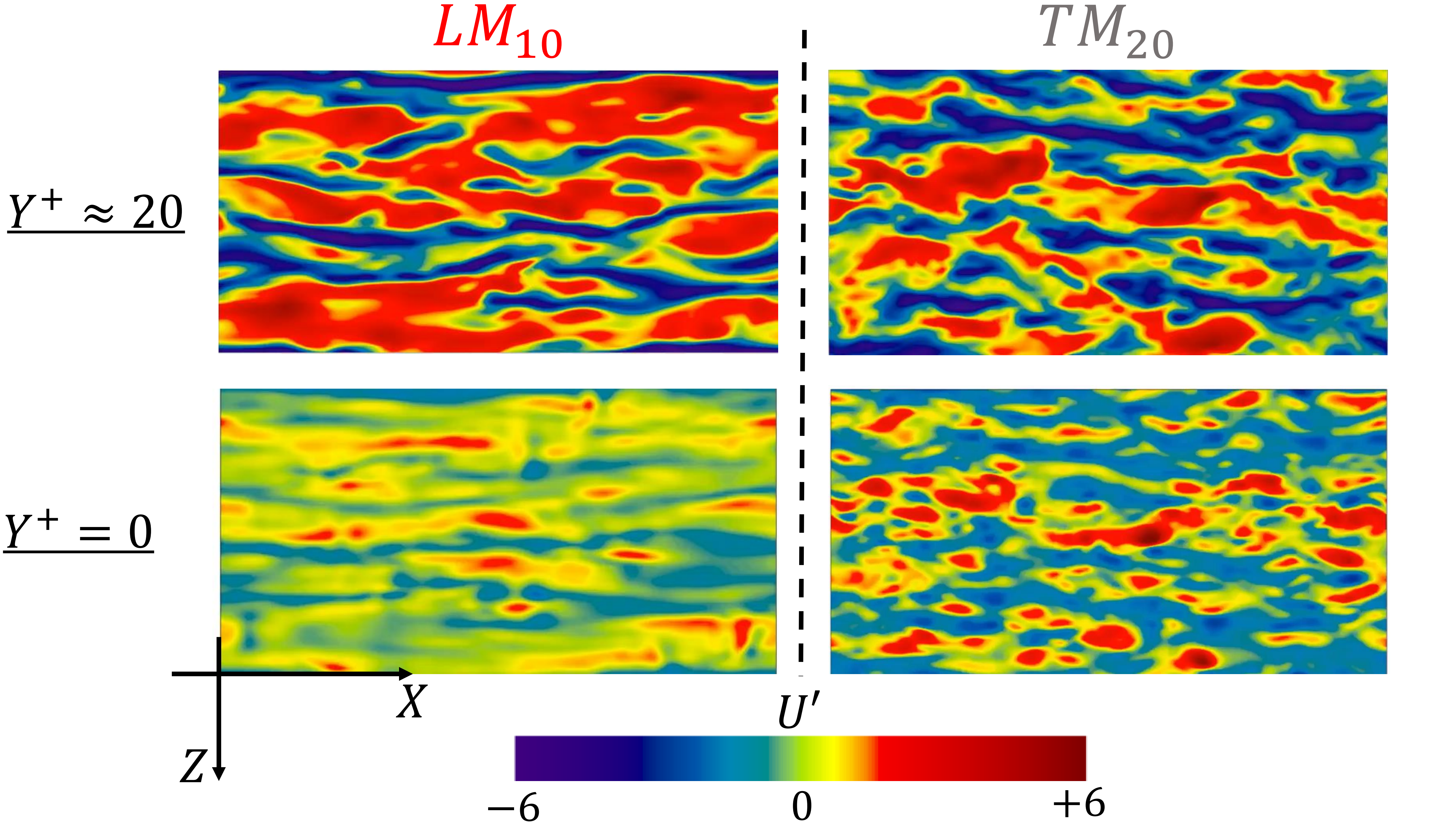
Root-mean-squares of velocity fluctuations

$$U_{rms} = \left(\overline{U'U'}\right)^{0.5}, V_{rms} = \left(\overline{V'V'}\right)^{0.5},$$
$$W_{rms} = \left(\overline{W'W'}\right)^{0.5}$$

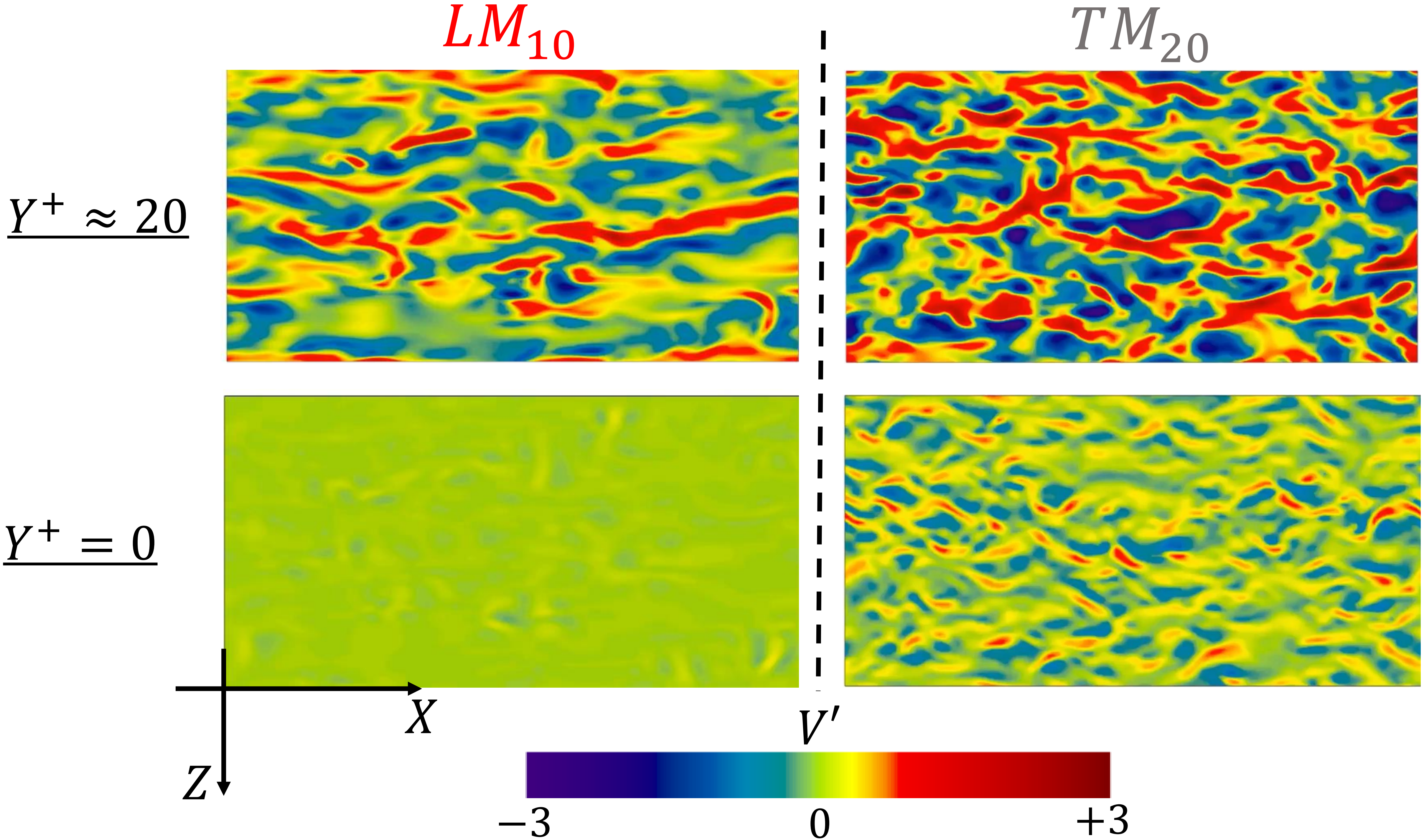


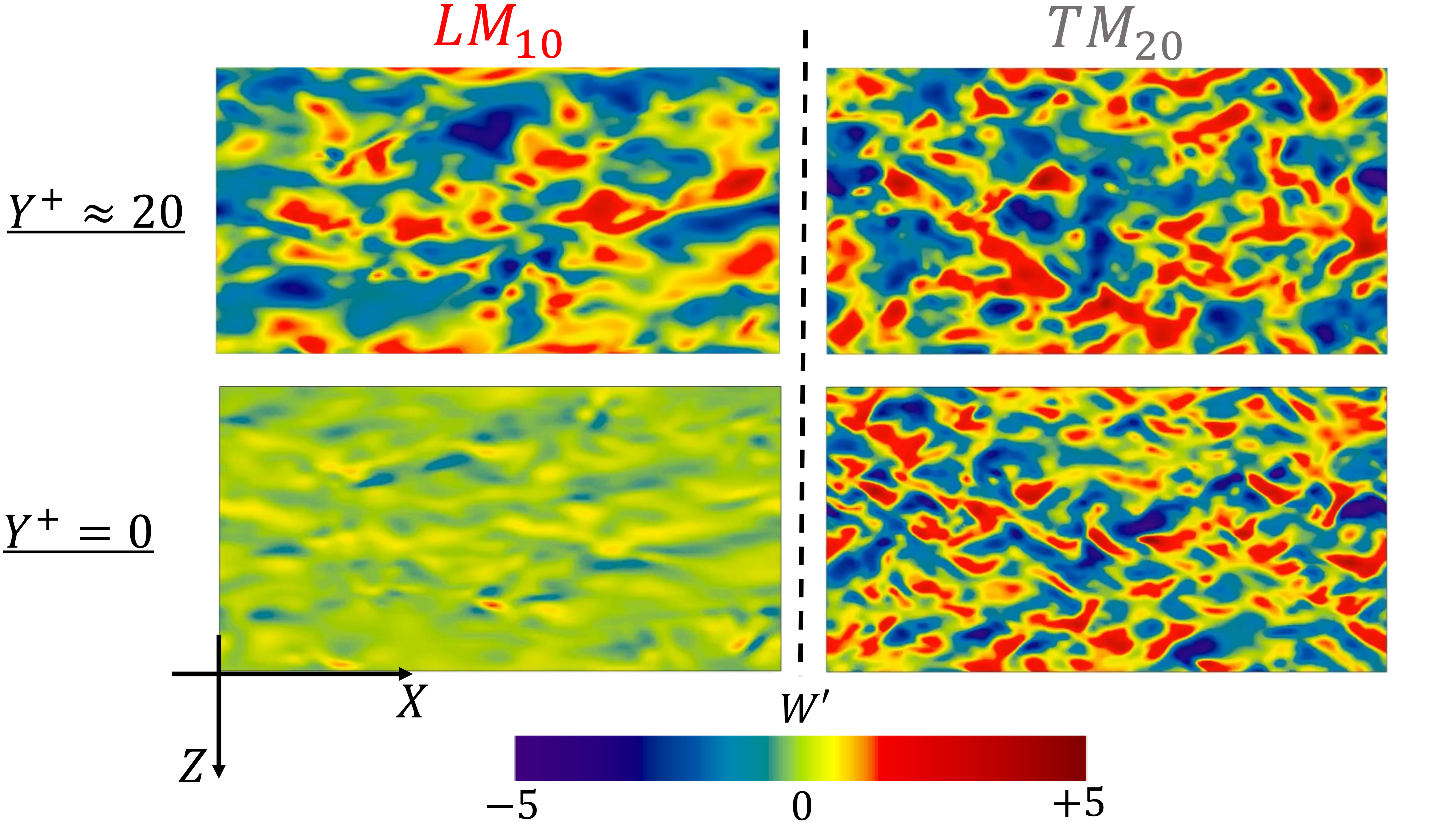




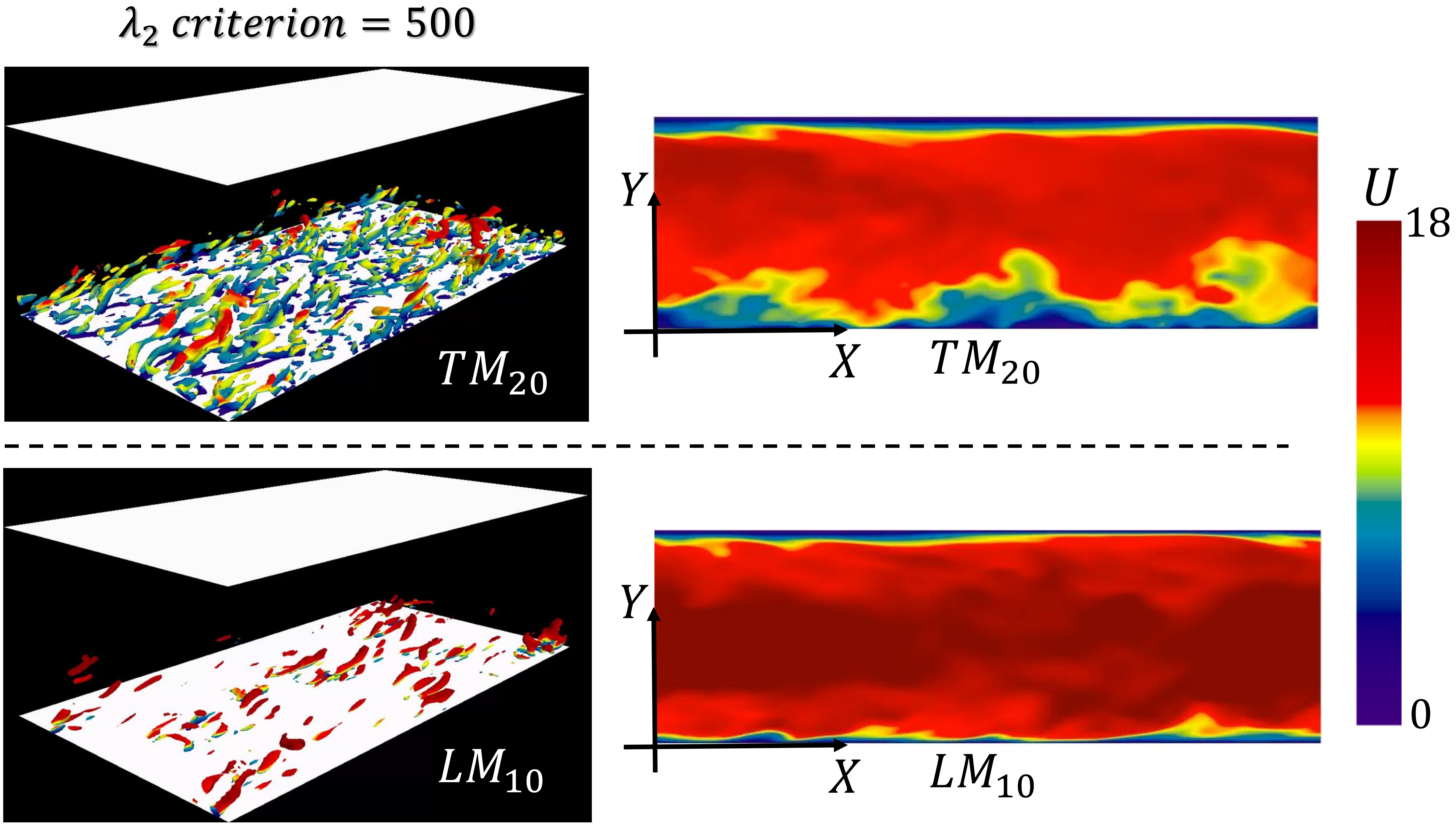


LM_{10}
TM_{20}
$Y^+ \approx 20$
$Y^+ = 0$
X
Z
U'
-6
0
$+6$


LM_{10}
TM_{20}
$Y^+ \approx 20$
$Y^+ = 0$
X
Z
V'
-3
0
$+3$



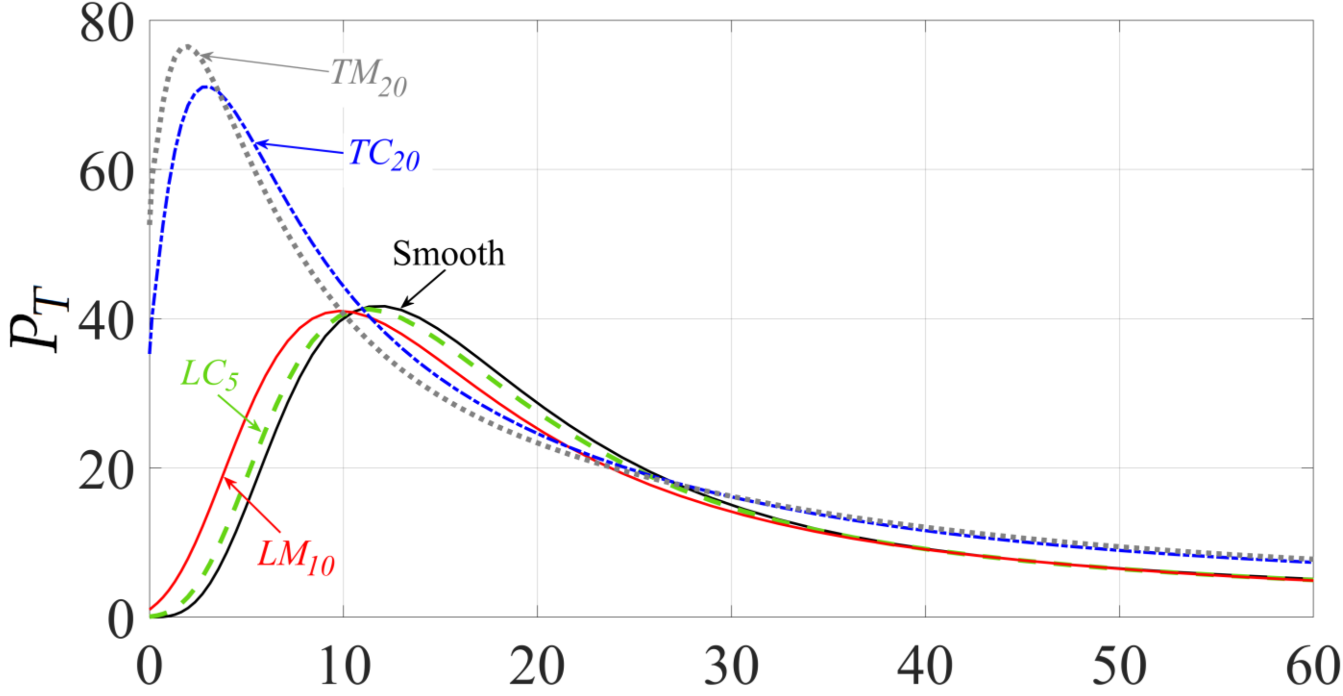
LM_{10}
TM_{20}
$Y^+ \approx 20$
$Y^+ = 0$
X
Z
W'
-5
0
$+5$



λ_2 criterion = 500
TM_{20}
Y
X
TM_{20}
U
18
0
LM_{10}
Y
X
LM_{10}

Production rate of TKE (normalized)

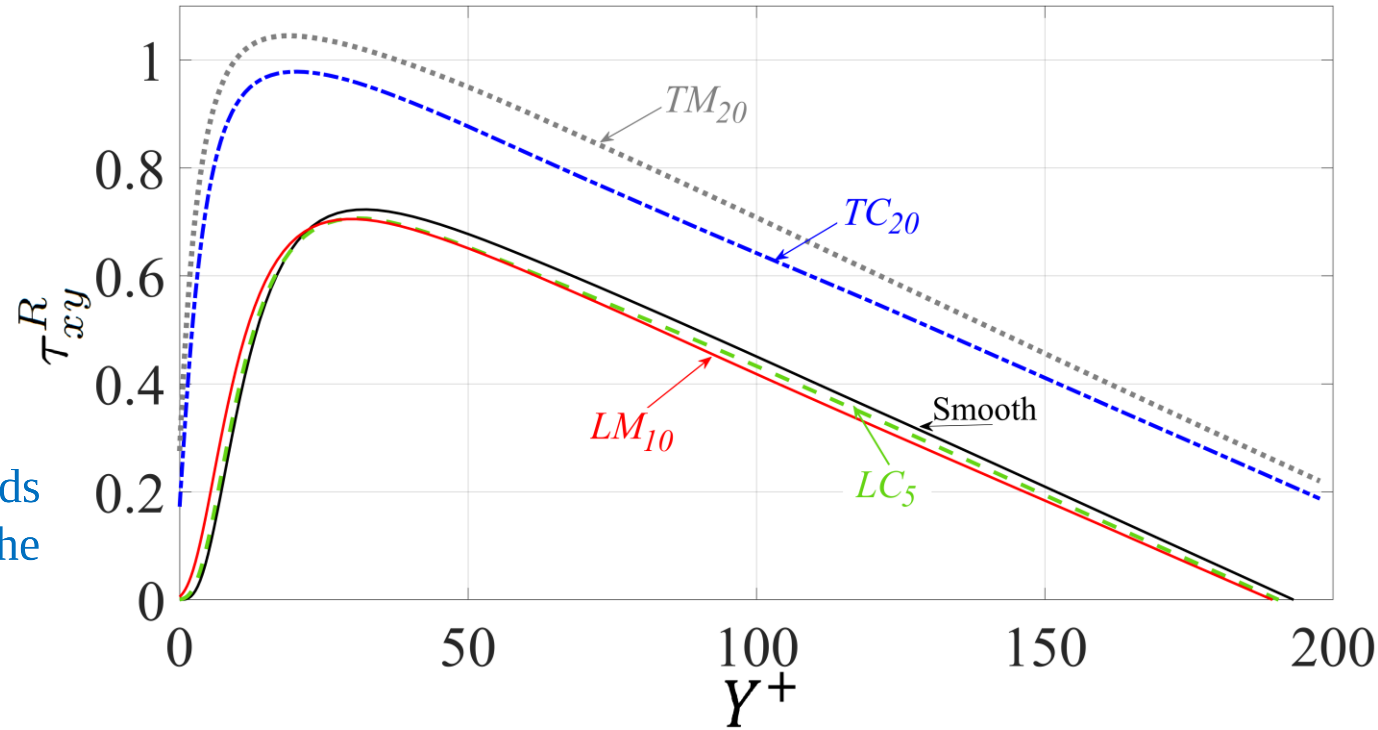
$$P_T = -\frac{1}{Re_\tau}\overline{U_i'U_j'}\frac{\partial \overline{U_i}}{\partial X_j}$$

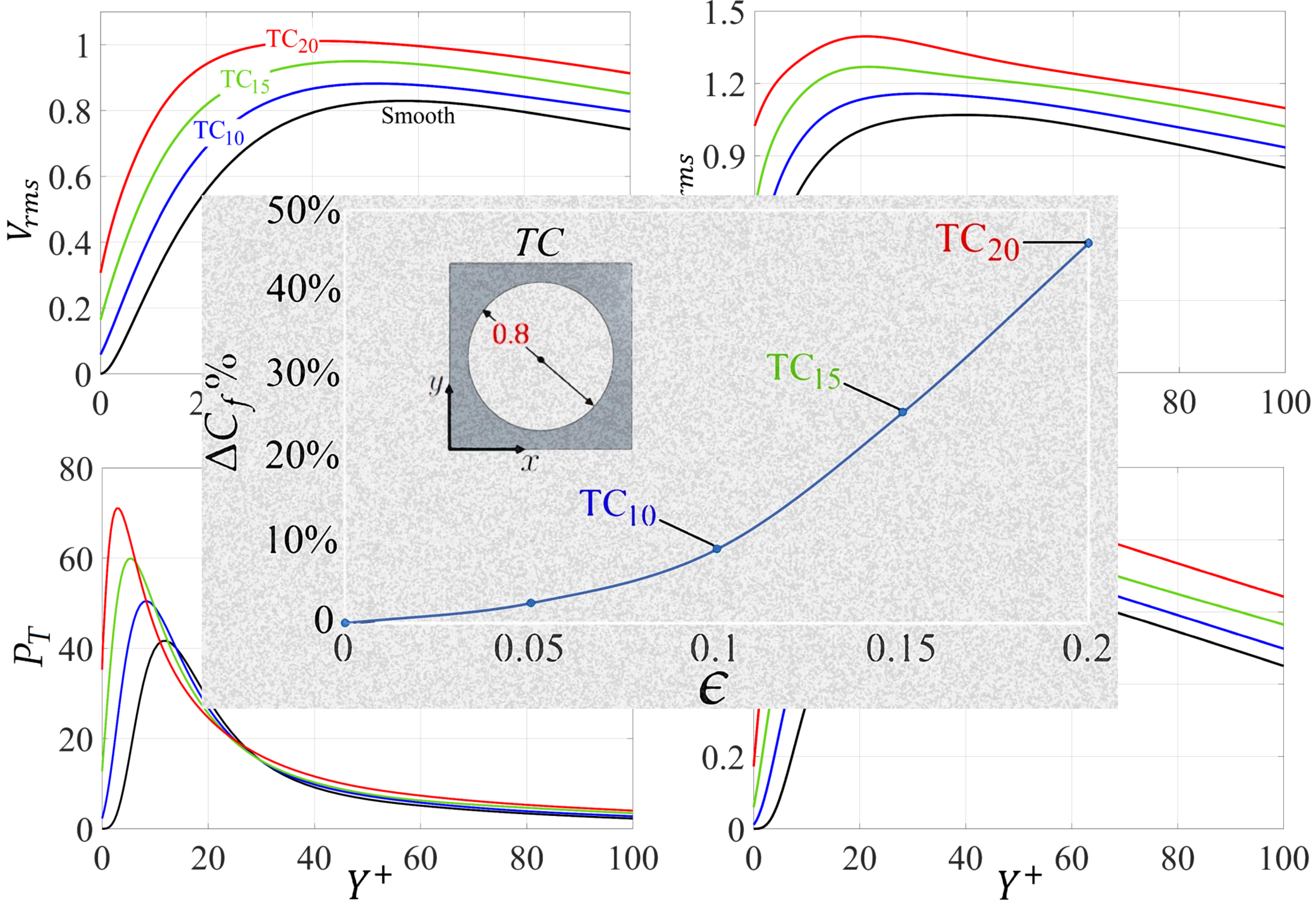


Reynolds shear stress (normalized)

$$\tau_{xy}^R = -\overline{U'V'}$$

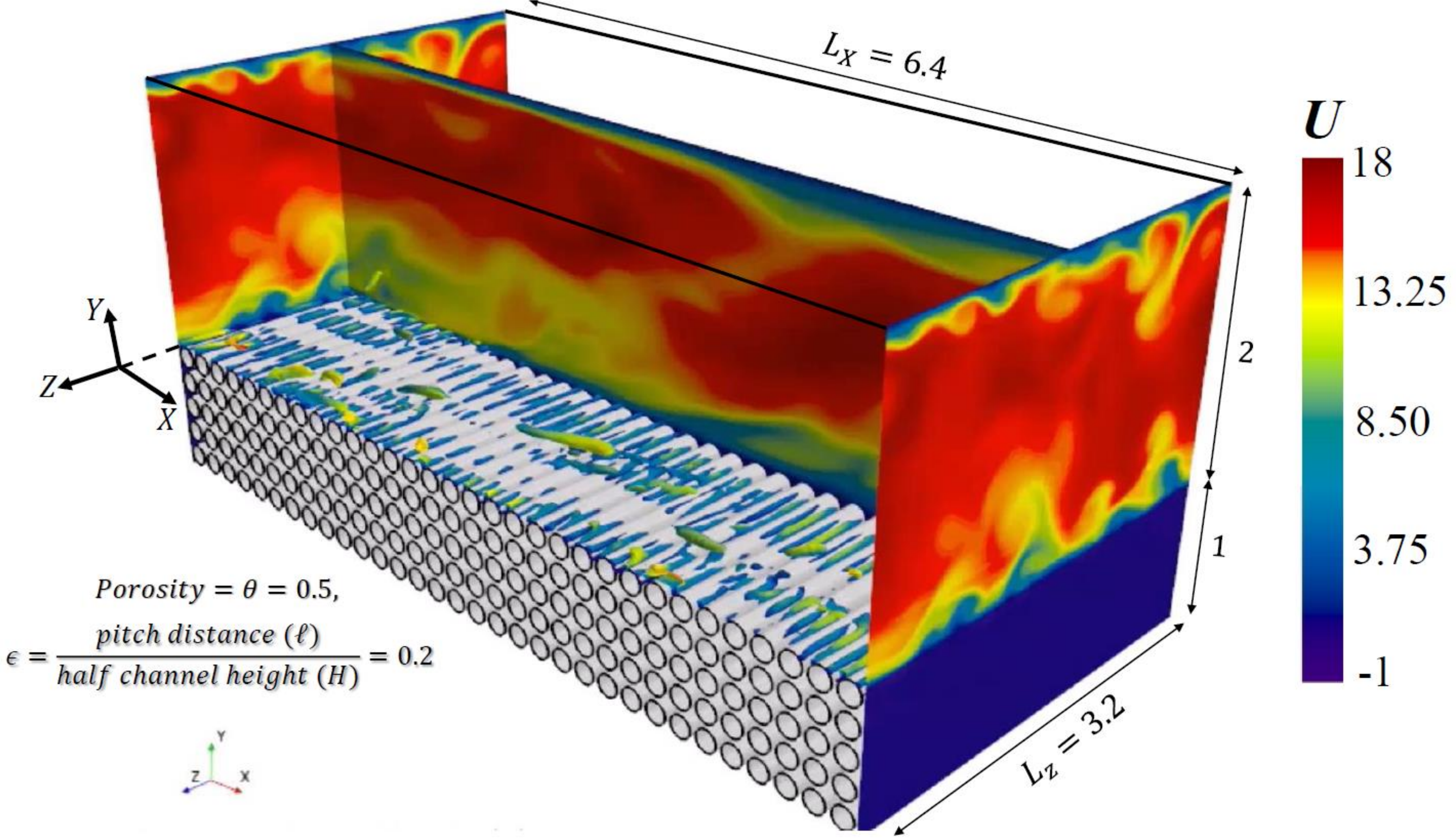
*Behaviors of the production rate of TKE and the Reynolds shear stress near permeable walls clearly interpret the adverse/favorable effects on skin-friction drag



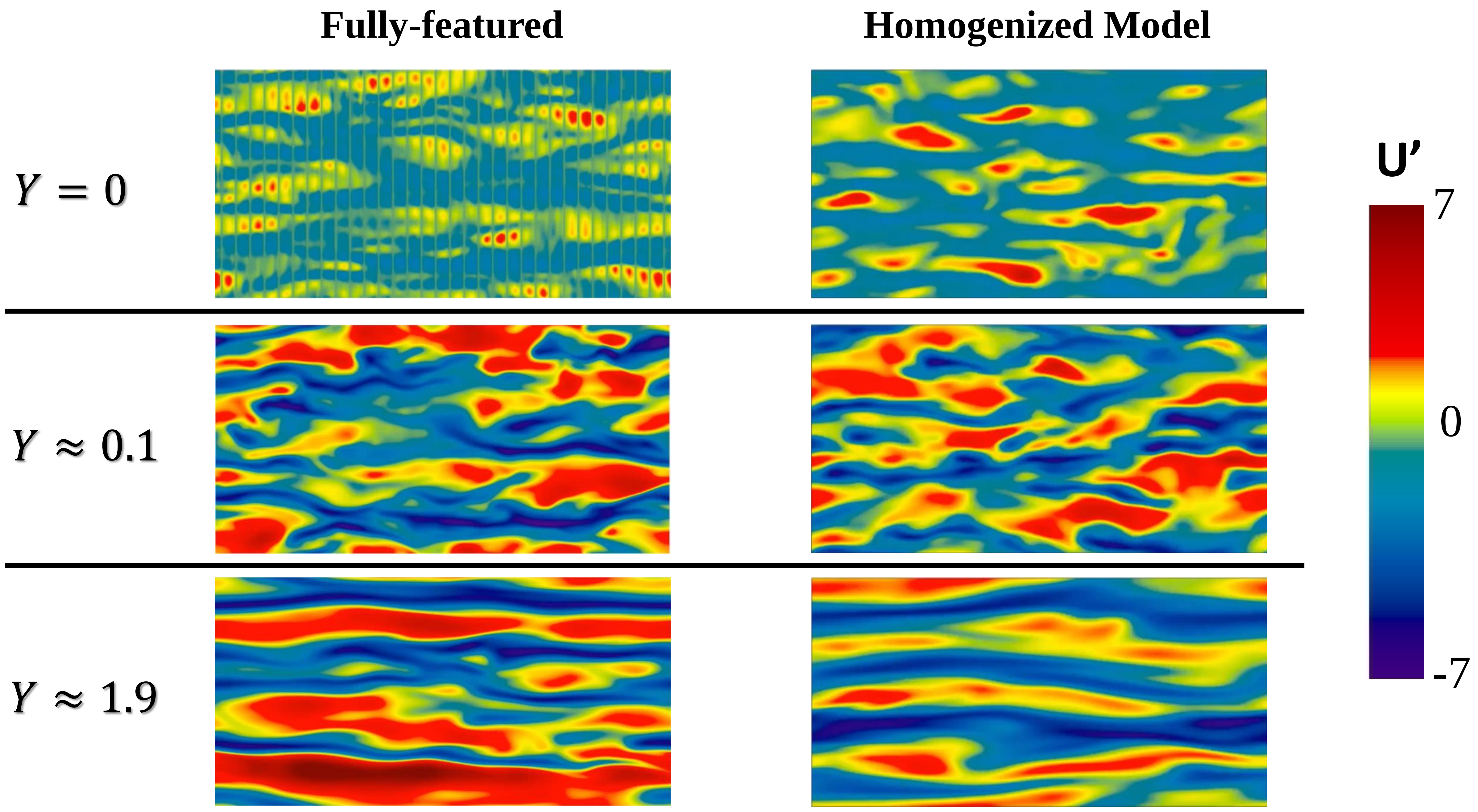


V_{rms}
TC$_{20}$
TC$_{15}$
TC$_{10}$
Smooth
Y^+
P_T
$\Delta C_f\%$
50%
40%
30%
20%
10%
0
0.05
0.1
0.15
0.2
ϵ
TC
0.8
y
x

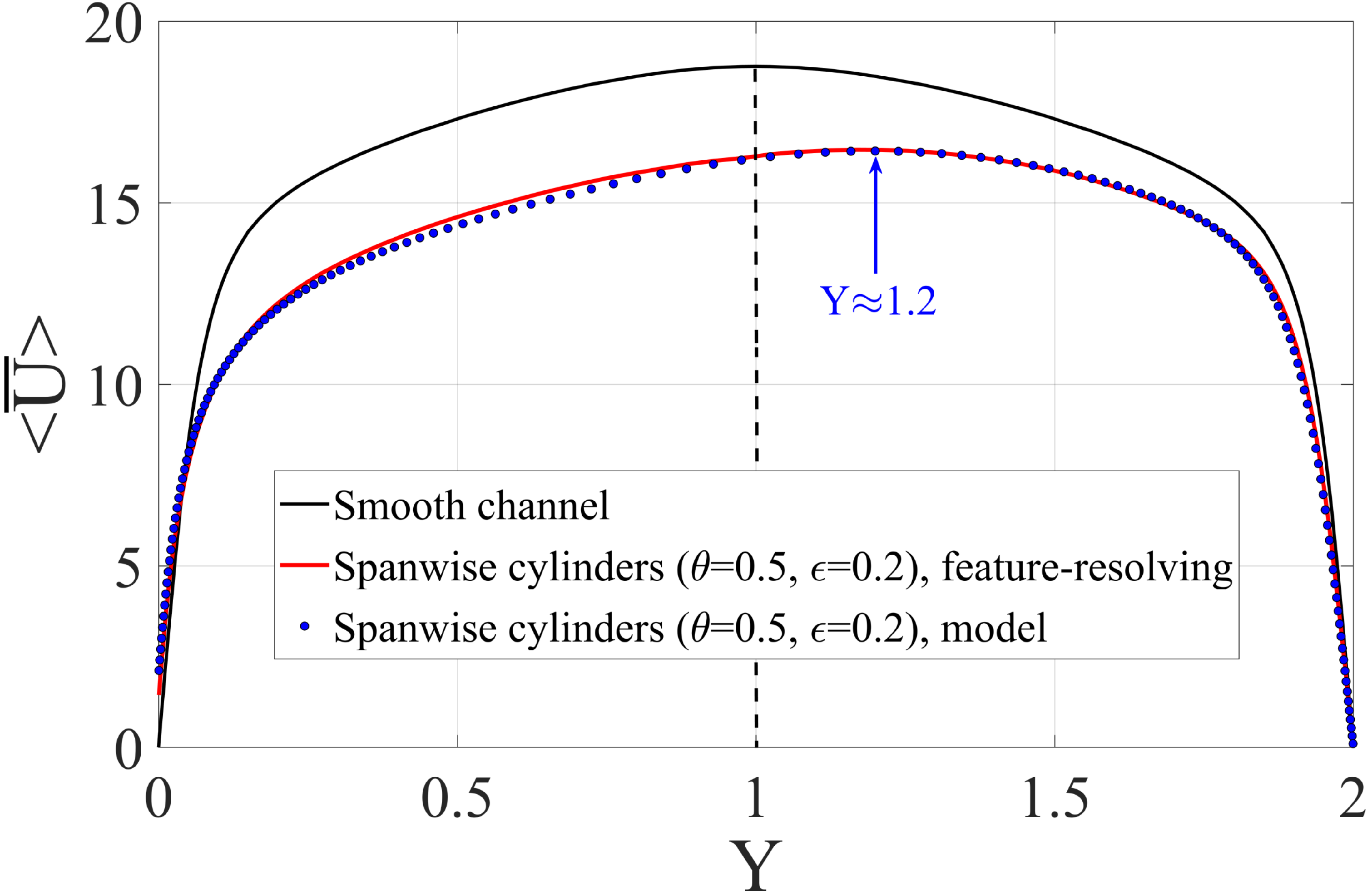
TC_{20}: Fully feature-resolving simulation



$L_X = 6.4$
U
18
13.25
8.50
3.75
-1
2
1
$L_Z = 3.2$
Y
Z
X
$Porosity = \theta = 0.5,$
$\epsilon = \frac{pitch\ distance\ (\ell)}{half\ channel\ height\ (H)} = 0.2$



Fully-featured
Homogenized Model
$Y = 0$
$Y \approx 0.1$
$Y \approx 1.9$
U'
7
0
-7



20
15
10
5
0
0
0.5
1
1.5
2
Y
$\langle \overline{U} \rangle$
Y≈1.2
Smooth channel
Spanwise cylinders (θ=0.5, ϵ=0.2), feature-resolving
Spanwise cylinders (θ=0.5, ϵ=0.2), model

Conclusions

- Properly engineered permeable substrates can reduce drag in wall-bounded turbulent flows by attenuating the near-wall coherent structures.

- In the homogenization approach followed, the flow is not resolved in the porous layer, but an effective velocity boundary condition is developed, and enforced, at a virtual interface between the porous bed and the channel flow.

- The implementation of the homogenization approach significantly reduces the numerical cost of direct numerical simulations over porous layers.

- The results, examined in terms of mean values and turbulence statistics, demonstrate the drag-reducing effects of porous substrates with streamwise-preferential alignment of the solid grains.